Nonhomogeneity of dusty crystals and plasma diagnostics

L. I. Podloubny,[1] P. P. J. M. Schram,[2] and S. A. Trigger[1,2]
[1]Institute for High Temperatures, Russian Academy of Sciences, Izhorskaya Street 13/19, 127412 Moscow, Russia
[2]Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

(Received 15 March 1999)

Real dusty crystals are inhomogeneous due to the presence of external forces. We suggest approximations for calculations of different types of inhomogeneous dust crystals (DC's) (chain and DC's with a few slabs) in the equilibrium state. The results are in a good agreement with experimental results and can be used as an effective diagnostic method for many dusty systems.
[S1063-651X(99)13210-0]

PACS number(s): 52.25.−b, 64.70.−p, 94.10.Nh

I. INTRODUCTION

Formation of dust crystals (DC's) takes place in a vertical electric field of the sheath, the gravitational field, and a horizontal electrical field. The external field, acting in vertical and horizontal traps, stabilizes the three-dimensional DC's of finite size and linear chains of d ions (horizontal traps for confinement of one-dimensional DC's are used in [1,2]). The pressure of the boundaries and the external field violates the translational invariance and leads to a dependence of the distances between nearest neighbors in the lattice of dust particles on the position of the particles (see Figs. 1 and 2). Therefore, macroscopic inhomogeneity in a lattice is a phenomenon not present in the usual infinite (very large) crystal.

Even in the approximation of central force for interparticle interaction between d ions, DC's possess a layered structure (the layered structure of usual atomic crystals, as graphite, is connected with the anisotropy of the interparticle interaction).

The vertical and horizontal distances between nearest neighbors (lattice constants $R_\parallel$ and $R_\perp$) are, in general, different functions of position in different directions from the center of the crystal (center of inertia). Deformation of DC's in the fields of the traps depends on its characteristics and on the plasma parameters. Therefore, the electrostrictional response of d-ion systems on a static external disturbance can be used as a diagnostic tool for DC's and the surrounding plasma. In particular, the charge Q of d ions, the screening length R_D, the concentration of the small ions, and the electric field in the sheath can be determined. In the present paper the possibility of using the inhomogeneity of DC's for plasma diagnostics is considered theoretically.

Recently dusty plasma diagnostics appear on the basis of investigations of the dispersion curves $\omega(k)$ for d ion sound [3] and properties of forced oscillations of linear d ion's chains in an electric field [1] and under the action of laser impulses [2]. The static diagnostic suggested in this paper is simpler for the theoretical description and experimental realization than the dynamic sounding considered in [1–3].

For the description of a lattice configuration of N d ions in a state of deformation under the influence of external gravitational and electric forces $\overline{f_n}=-\nabla V_n$ and interparticles forces $\overline{F_n}=-\nabla U_n$, we will use the balance equations. Here V_n is the potential energy of the d ion with number n and U_n is the potential energy of the interaction between the d ion with number n and all other ones. We do not take into account the force connected with momentum transfer from the small ions to the d ions. This force very often can be omitted because in the case where it is essential the d ions can be found not only below the sheath, but also on top of it, which is not observed in the experiment discussed below.

We will use the simple approximation of nearest neighbors for the description of interparticle interaction. This approximation apparently gives a good picture of the inhomogeneity of DC's under the influence of external forces and with a screening length $R_D \sim R_\perp, R_\parallel$. We also will neglect a possible dependence of the d ion charge Q on the location in the inhomogeneous DC's (with respect to d ion density). Therefore, we suggest $Q=\text{const}$ in our considerations.

II. EQUATIONS OF STATIC EQUILIBRIUM

For the case of inhomogeneous three-dimensional DC's we will use a simple quasi-one-dimensional model of DC's, in which the layer lattice with a real potential is changed into a one-dimensional vertical chain of particles. The effective potential for this model can be calculated by integration of the interaction with the nearest layer with distributed charge $\sigma=Q/S_0$ (S_0 is the surface for a d ion in horizontal direction),

$$\langle U(r)\rangle_{xy}=\frac{2\pi}{S_0}\int_0^\infty d\rho\rho U(\sqrt{z^2+\rho^2}). \tag{1}$$

For the Debye-Hueckel interaction and simple hexagonal lattice potential (1) has the form

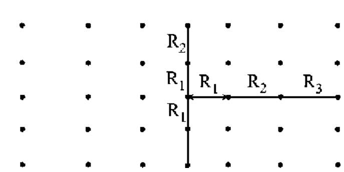

FIG. 1. An example of inhomogeneous plane dusty crystals (2D crystal).

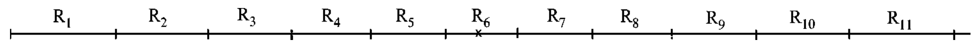

FIG. 2. Linear chain of dust particles (number of particles $N=12$) in a parabolic trap. The inhomogeneity was calculated on the basis of Eq. (15) for $t=0.18$ and compared with the experimental data [1].

$$U(z)=\left\langle \frac{Q^2}{2}e^{-\kappa r}\right\rangle_{x,y}=U_0 e^{-\kappa z},\quad \kappa=\frac{1}{R_D},\qquad U_0=2\pi\frac{Q\sigma}{\kappa},\quad \sigma=\frac{2Q}{\sqrt{3}R_\perp^2}. \tag{2}$$

This model permits us to calculate the dependence of the distances between the nearest slabs $R_n(z)$ as a function of height.

In the general case of pair interaction between the d ions in the external electric and gravitational fields of the sheath the potential energy can be written in the form

$$U+V=\sum_{k=1}^{N-1}U_k+\sum_{k=1}^{N}V_k,\quad U_k=U(R_k),\quad V_k=V(z_k),\quad R_k=z_{k+1}-z_k. \tag{3}$$

Here we take into account only the interaction between neighboring particles. The potential energy for a horizontal chain of N interacting d ions in the electric field of the trap has an analogous form and stabilizes this chain in the x direction ($z_k \to x_k$). The conditions of balance of external and internal forces lead to a system of equations that determines the configuration of the d ions:

$$\begin{cases} U_k'-U_{k+1}'+V_{k+1}'=0,\quad k=1,2,3,\dots N-2 \quad U_k'=\dfrac{dU}{dR_k},\quad V_k'=\dfrac{dV}{dz_k},\\ -U_1'+V_1'=0,\\ U_{N-1}'+V_N'=0. \end{cases} \tag{4}$$

Summation of the left-hand parts of these equations leads to the obvious condition of a zero sum of the external fields: $\Sigma_{k=1}^{N}V_k'=0$.

For the stabilization of horizontal chains an external field in the form of a parabolic well in the chain direction has been used in [1,2].

$$V_k=\frac{1}{2}m\omega_0^2(x_n-X_0)^2,\quad X_0=\frac{1}{N}\sum_{k=1}^{N}x_k, \tag{5}$$

Here X_0 is the center of inertia for a chain and ω_0 is a parameter that determines the shape of the pit. According to [4] the vertical electric field in a sheath changes linearly with the height. This dependence is realized approximately in the regions not too close to the lower electrode and the border of the presheath: the quadratic approximation for the potential $\varphi(z)$ in the plasma layer is also used in [5,6] for the analysis of the equations of motion of DC's. Therefore, in the case of a vertical potential well we use in Eq. (4) the expansion

$$V(z_k)=mgz_k+Q\varphi_0+Q\varphi_0'(z_k-X_0)+\frac{1}{2}\omega_0^2(z_k-X_0)^2,$$
$$\varphi_0=\varphi(X_0),\quad \varphi_0'=\varphi'(X_0),\quad m\omega_0^2=Q\varphi''(X_0). \tag{6}$$

The parabolic approximation for the vertical electric field is reasonable for the case of sufficiently thin DC's. To estimate the maximal thickness $\ell=z_N-z_1=2(X_0-z_1)$ of DC's, for which this approximation is true, let us consider $\varphi(z)=\varphi(0)\exp(-z/R_D)$ and use the condition

$$\frac{1}{3}|Q\varphi_0'''|(X_0-z_1)^2\sim\frac{1}{3}|Q\varphi_0'|\left(\frac{\ell}{2R_D}\right)^2<Q\varphi_0''(X_0-z_1)$$
$$\sim|Q\varphi_0'|\frac{\ell}{2R_D}.$$

Then the necessary inequality is $\ell<6R_D$, which is usually satisfied (see, for example, [3,5,6]). The linear terms in Eq. (6) are really absent because of the condition of zero total external forces:

$$mg+Q\varphi'(X_0)=0. \tag{7}$$

This condition determines the position of the center of inertia for the system of levitated d ions.

By use of parabolic approximation (6) in balance equations (4) and subtracting from each equation the previous one, we find

$$\begin{cases} 2U_k'-U_{k+1}'-U_{k-1}'+m\omega_0^2R_k=0,\quad k=2,3,\dots N-2\\ 2U_1'-U_2'+m\omega_0^2R_1=0,\\ 2U_{N-1}'-U_{N-2}'+m\omega_0^2R_{N-1}=0. \end{cases} \tag{8}$$

As follows from Eq. (8) the intervals R_k are symmetric with respect to the center:

$$R_1=R_{N-1},\quad R_2=R_{N-2},\dots,\quad R_k=R_{N-k},\dots.$$

III. STRUCTURE OF DC's WITH AN ATTRACTIVE (FOR LARGE DISTANCES) AND WITH A PURELY REPULSIVE POTENTIAL

According to Eqs. (4) and (8), for isolated systems of d ions ($V_k'=0$), there are two different possibilities when external fields are absent.

If the pair interaction between d ions is a nonmonotonic function and leads to repulsion at small distances and to attraction at large distances, then the solution of Eq. (4) reads

$$U_1'=U_2'=\cdots=U_{N-1}'=0. \tag{9}$$

This solution describes a homogeneous chain of N d ions with equal distances between nearest neighbors $R_1=R_2=\cdots=R_{N-1}=R_0$. The potential energy has a minimum for this state. In this case the weakly inhomogeneous configurations of d ions with external force $V_k'\neq 0$ can be described on the basis of small deformations $|R_0-R_k|\ll R_0$. Then we use the expansion

$$U(R_k)=U_0+\frac{m\Omega^2}{2}(R_k-R_0)^2, \quad m\Omega^2=U''(R_0). \tag{10}$$

If the pair interaction $U(R_k)$ has a monotonic purely repulsive form, the d ions of an isolated system are unstable and, according to Eq. (9), all R_k are infinite. In this case stabilization of the system in a weak external field playing the role of a trap leads also to a slightly inhomogeneous state, in which the deviations of the intervals from the average are small,

$$R_0=\frac{1}{N-1}\sum_{k=1}^{N-1}R_k, \quad |R_0-R_k|\ll R_0. \tag{11}$$

In this case the alternative quadratic expansion of the energy for d-d interactions has the form

$$\begin{aligned}\sum_{k=1}^{N-1}U_k&=(N-1)U_0+U_0'\sum_{k=1}^{N-1}(R_k-R_0)\\&\quad+\frac{1}{2}m\Omega^2\sum_{k=1}^{N-1}(R_k-R_0)^2\\&=(N-1)U(R_0)-(N-1)U_0'R_0+U_0'(z_N-z_1)\\&\quad+\frac{1}{2}m\Omega^2\sum_{k=1}^{N-1}(R_k-R_0)^2,\end{aligned}$$

$$U_0=U(R_0), \quad U_0'=\frac{dU(R_0)}{d(R_0)},$$

$$m\Omega^2=\frac{d^2U(R_0)}{dR_0^2}, \quad \sum_{k=1}^{N-1}R_k=z_N-z_1. \tag{12}$$

For a potential with a well $U'(R_0)=0$ expansions (10) and (12) coincide; therefore, small deformations $s_k=R_0-R_k$ of the system in an external field can then be described by the general equations of force balance:

$$\begin{cases}2\cosh t s_k-s_{k+1}-s_{k-1}-\dfrac{\omega_0^2}{\Omega^2}R_0=0, \quad \cosh t=1+\dfrac{\omega_0^2}{2\Omega^2}, \quad k=2,3,\ldots,N-2\\[2mm] 2\cosh t s_1-s_2-\dfrac{\omega_0^2}{\Omega^2}R_0-\dfrac{U_0'}{m\Omega^2}=0,\\[2mm] 2\cosh t s_{N-1}-s_{N-2}-\dfrac{\omega_0^2}{\Omega^2}R_0-\dfrac{U_0'}{m\Omega^2}=0.\end{cases} \tag{13}$$

Here, for purely repulsive interaction R_0 is the average. For the case with attraction $U_0'=0$ and R_0 is the equilibrium distance in the isolated system of d ions.

IV. SOLUTIONS AND NUMERICAL RESULTS

A general solution of the equations in finite differences (13) can be obtained in the form

$$s_k=R_0-R_k=R_0+Ae^{kt}+Be^{-kt}. \tag{14}$$

Taking into account the symmetry of the system $s_k=s_{N-k}$, the connection between the coefficients $B=Ae^{Nt}$ can be found. The coefficient A can be found from the boundary condition for $k=1$ (or for $k=N-1$). Finally, for the interval number k and purely repulsive potential we find

$$\begin{aligned}R_k&=\left(R_0-\frac{U'(R_0)}{m\Omega^2}\right)\frac{\cosh\left(\dfrac{N}{2}-k\right)t}{\cosh\dfrac{Nt}{2}}\\&=\left(R_0-\frac{U'(R_0)}{m\Omega^2}\right)\frac{C_{k-1}'(\cosh t)+C_{N-k-1}'(\cosh t)}{C_{N-1}'(\cosh t)}.\end{aligned} \tag{15}$$

Here, $C_n'(x)$ are the Gegenbauer polynomials. For the case of interaction with attraction $U'(R_0)=0$, the intervals R_k have the form

$$R_k = R_0 \frac{\cosh\left(\frac{N}{2}-k\right)t}{\cosh\frac{N}{2}t}. \tag{16}$$

Therefore, in the parabolic trap formed by the external forces, a chain of d ions is compressed symmetrically with respect to the center of inertia, and the central regions more strongly than the ones outwards $R_1 > R_2 > \dots$. For the resulting electrostrictional reduction of the length ℓ of an entire chain it follows from Eq. (16) that (R_0 is the equilibrium distance in a homogeneous chain)

$$\ell = \sum_{k=1}^{N-1} R_k = 2R_0 \frac{\cosh\frac{t}{2}\sinh\frac{N-1}{2}t}{\sinh t \cosh\frac{Nt}{2}} < (N-1)R_0. \tag{17}$$

For sufficiently long ($N \gg 1$) horizontal chains and for (in vertical direction) quasi-one-dimensional dusty crystals the profile distributions of charge density and mass and thereby the ''constants'' of the elastic forces can be obtained in the approximation of continuous media by use of Eqs. (16) and (17). The surface density of charge is proportional to the mass density and, therefore, there is balance of the external volume electric and gravitational forces in each point of a horizontal plane at fixed height. This means that even inhomogeneous planes (Fig. 1) and horizontal chains (Fig. 2), which are more dense in the center, are not suspended in the central part of the dusty system, where the density is higher. Enlargement of the density in the center of horizontal crystalline planes is observed in the experiments [7], but quantitative measurements are unknown to us. Parallel to oscillation and wave measurements in horizontal chains, the equilibrium positions of d ions have also been determined in the electric field of a horizontal trap [1,2,8]. According to the data of these papers for the case $N=12$ the ratios of the intervals between neighboring d ions in the direction of the center are $R_1 : R_2 : \dots R_6 = 1.44{:}1.22{:}1.11{:}1.05{:}1.01{:}1.00$. These results are reasonably described by formula (15), in which for $t=0.18$ (and correspondingly $\omega_0 = 0.2\Omega$) these ratios are 1.43:1.27:1.15:1.07:1.02:1.00.

The experimental data for the other half of the chain $R_6 : R_7 : \dots R_{11} = 1{:}1.01{:}1.01{:}1.08{:}1.20{:}1.32$ agree less with our theory for the (with respect to the center) symmetric chain and they are essentially different from the experimental data for the first half of the chain. We think that this asymmetry is a consequence of the asymmetric and not exactly parabolic $V(x) \approx \frac{1}{2} m\omega_0^2 x^2$ shape of the external electric field (here x is the distance from the center of the chain). According to [8] $m\omega_0^2 = 2.55 \times 10^{-11}\,\mathrm{kg/s^{-2}}$ and $m = 6.73 \times 10^{-13}$ kg. Using the data on the equilibrium configuration R_n and the parameter of the trap $\omega_0 = 6.15\,\mathrm{s}^{-1}$ we find the important characteristic of $d-d$ interaction $\Omega = 5\omega_0 = 30.7\,\mathrm{s}^{-1}$.

For the chain with $N=4$ the experimental data, according to [1,8], give $\omega_0 = 6.25\,\mathrm{s}^{-1}$ and $R_1 = 1989\,\mu\mathrm{m}$, $R_2 = 1910\,\mu\mathrm{m}$, and $R_3 = 2031\,\mu\mathrm{m}$. The average interval $R_\perp = 1960\,\mu\mathrm{m}$ for the case $N=4$ is twice as large as $R_\perp = 10^3\,\mu\mathrm{m}$ for the chain with $N=12$. This is probably connected with the higher charges (almost three times) of the d ions in [1] and therefore with the stronger repulsion between them at the partially same compressing external field of the horizontal trap. For the conditions of the experiments [1] the asymmetry of the external field, connected with the nonquadratic form of the potential $V(x)$ is still stronger than in [2] and this was the reason to use for the bordering intervals the expression $\langle R_1 \rangle = (R_1 + R_3)/2 = 2010\,\mu\mathrm{m}$. Then according to Eq. (10) we have $\langle R_1 \rangle / R_2 = 1 + \omega_0^2/2\Omega^2 = 1.05$ and $\Omega = 3.16\omega_0 = 19.7\,\mathrm{s}^{-1}$.

It is necessary to emphasize that all the results for the case $N=4$ and $N=12$ are applicable for both cases: purely repulsive d-d interaction and d-d interaction with an attractive part, because, as follows from Eqs. (15) and (16), the ratios of intervals R_k are the same in these cases.

The known experimental data on equilibrium intervals $R_\parallel$ between neighbouring ions in vertical traps concern only dust crystals with two horizontal crystalline planes ($N=2$) and have been obtained in [5,6]. In [3] a dust crystal with $N=3$ has been investigated but the thickness of the crystal was not measured.

According to [5,6] the ratio $(R_0 - R_\parallel)/R_0 = 0.2$ and does not depend on the ion's mass.

In [5] experiments are reported with dust crystals, formed by d ions with radii 4.7 μm and 2.4 μ, which leads to a difference of gravitational force proportional to $m_1/m_2 \simeq 8$. The position of the center of inertia X_0 of the dust crystal must be considerably changed in this case: a lighter crystal will shift over a distance $\sim R_D$, as follows from Eq. (7). A measurement of this effect was not reported in [5].

According to Eq. (16),

$$1 - \frac{R_\parallel}{R_0} = \frac{\omega_0^2}{\omega_0^2 + 2\Omega^2} = 0.2 \tag{18}$$

and, therefore, $\Omega = \sqrt{2}\omega_0$. In contrast with [1,2] the parameter $\omega_0^2 = (1/m) V_0''(x)$ for the vertical electric field in a sheath is here unknown. It can be determined only on the basis of knowledge of the interaction potential between the d ions, via the parameter $\Omega^2 = (1/m) U''(R_\parallel)$.

Purely repulsive interaction (1) leads to another result. For $N=2$ the exact system of balance Eqs. (3) has the form

$$\begin{cases} -U'(R_\parallel) + V'(z_1) = 0, \quad R_\parallel = z_2 - z_1, \quad X_0 = \dfrac{z_1+z_2}{2}, \\ U'(R_\parallel) + V'(z_2) = 0, \quad z_{2,1} = X_0 \pm \dfrac{1}{2} R_\parallel. \end{cases} \tag{19}$$

In this case a more general model for external potential (6) than the linear one can be used for the description of the electric field in a sheath. Let us take

$$E(X_0 \pm \tfrac{1}{2} R_\parallel) = E(X_0) e^{\pm 1/2 \kappa R_\parallel}. \tag{20}$$

For the position of the center of inertia we have

$$mg = QE(X_0) \cosh \tfrac{1}{2} \kappa R_\parallel, \tag{21}$$

and according to Eq. (19) with $U(R_\parallel)$ taken from Eq. (2) we find

$$V'(z_2)-V'(z_1)=2QE(X_0)\sinh\frac{1}{2}\kappa R_\parallel=2\frac{4\pi Q^2}{\sqrt{3}R_\perp^2}e^{-\kappa R_\parallel}. \tag{22}$$

By eliminating $E(X_0)$ from these equations we finally find

$$\tanh\frac{\kappa R_\parallel}{2}=\alpha e^{-\kappa R_\parallel},\quad \alpha=\frac{4\pi Q^2}{\sqrt{3}R_\perp^2 mg}. \tag{23}$$

Using the experimental data [5] for m, Q, and $R_\perp=450\,\mu$m, $R_\parallel=360\,\mu$m we estimate the Debye radius $R_D=973\,\mu$m. In the case of a dust crystal with a lower m, Q and $R_\perp=350\,\mu$m, $R_\parallel=280\,\mu$m (see also [5]) we find $R_D=933\,\mu$m. Therefore, the Debye length R_D is approximately of the order of the interval between nearest d ions, which is in agreement with the estimates of [6].

For the electric field in a sheath we find from Eq. (21) $E(X_0)=2.84\times 10^3$ V/m^{-1} and $E(X_0+\Delta)=1.99\times 10^3$ V/m^{-1} for the case of d ions with radii 4.7 μm and 2.4 μm.

Let us neglect small changes of Debye radius R_D and take

$$\frac{E(X_0)}{E(X_0+\Delta)}=\exp\left(\frac{\Delta}{\langle R_D\rangle}\right)=1.42,\quad \langle R_D\rangle=950\,\mu\text{m}. \tag{24}$$

Then we obtain for the shift upwards Δ of the lighter crystal,

$$\Delta=0.35\langle R_D\rangle=332\,\mu\text{m}. \tag{25}$$

The moving of a dust crystal inside the sheath can be observed by different microgravity experiments (see some discussion, for example, in [9]).

We suggest here some experiments in which the properties of DC's can be studied under conditions of microgravity and even changing gravity.

One of these experiments (under terrestrial conditions) can be performed in a horizontal discharge, where in the horizontal direction there is only an electric force and momentum transfer from the small ions to the dust particles. For such an experiment the latter force can be very essential in contrast to the conditions considered in this paper.

The second group of experiments is connected with the effective gravity created in space stations by rotation of dusty plasma. If h and $g_{\rm eff}$ are the distance from the axis of rotation to the negative electrode and the acceleration of the center of inertia for the dusty system, respectively, the obvious connection is given by

$$g_{\rm eff}=\omega^2(h-X_0). \tag{26}$$

For $g_{\rm eff}=g$ and $h=1$ m (rotation of the container inside the space station or rocket) or $h=10$ m (rotation of the space station as a whole) we find $\omega=3\,\text{s}^{-1}$ and $\omega=1\,\text{s}^{-1}$, respectively, which are conditions of a weakly inhomogeneous ($h\gg R_k$), artificial gravitational field where our results, obtained above, are applicable. Measuring the dependence $X_0=X_0(\omega)$ would permit us to investigate the profile of the electric field in a sheath and other characteristics of the dusty system and plasma. Of special interest is the investigation of the deformation of DC's in an essentially inhomogeneous rotation field ($h-X_0\sim 0.05$ m and $\omega\sim 15\,\text{s}^{-1}$). A detailed consideration of such experiment will be given in a separate paper.

In the case of a dust crystal with three horizontal crystalline planes the static equilibrium is described by the system of Eqs. (4) with $N=3$. In the approximation for the electric fields used before the coordinate of the average d ions z_2 and the value E_0 can be eliminated on basis of the balance of the external fields:

$$3mg=QE_0\sum_{n=1}^{3}e^{-\kappa z_n}=QE_0e^{-\kappa z_2}(1+e^{\kappa R_1}+e^{-\kappa R_2}). \tag{27}$$

For the system of equations determining the vertical intervals R_1 and R_2, we obtain

$$\frac{\alpha}{3}(e^{-\kappa R_1}-2e^{-\kappa R_2})+\frac{1-e^{-\kappa R_2}}{1+e^{\kappa R_1}+e^{-\kappa R_2}}=0,$$

$$\frac{\alpha}{3}(e^{-\kappa R_2}-2e^{-\kappa R_1})+\frac{e^{\kappa R_1}-1}{1+e^{\kappa R_1}+e^{-\kappa R_2}}=0. \tag{28}$$

Even for the highest pressure of neutrals in [3], $p=300$ mTorr ($Q=7.2\times 10^3\,e$, $R_\perp=0.28$ mm, and $\kappa R_\perp=0.61$) the parameter $\alpha/3=0.053\ll 1$. Suggesting $R_1=R_2=R_\parallel$ and $\kappa R_\parallel\ll 1$ it follows from Eq. (28) that

$$\kappa R_\parallel\approx\frac{\alpha}{1+\alpha}=0.14. \tag{29}$$

Let us emphasize that the vertical compression is symmetric ($R_1=R_2$) with respect to the central plane only for $\kappa R_\parallel\ll 1$. In contrast to the approximate Eqs. (8) for the parabolic wells, the exact Eqs. (28) are not symmetric for the interchange $R_1\rightleftarrows R_2$.

From Eqs. (27)–(29) with the Debye radius given in [3] it follows that vertical compression is important:

$$\frac{R_\perp-R_\parallel}{R_\perp}=0.77. \tag{30}$$

In the framework of the quadratic approximation for the potential energy of the system with $N=3$ we find according to Eqs. (15) and (16)

$$\frac{R_\perp-R_\parallel}{R_\perp}=\frac{\omega_0^2}{\omega_0^2+\Omega^2}. \tag{31}$$

Unfortunately, the vertical interval $R_\parallel$ has not been measured in [3]. The experimental data obtained in [3] are not sufficient to choose which variant is preferable for purely repulsive interaction or interaction with an attractive part: the quadratic model or the more exact description, Eqs. (27) and (28).

V. CONCLUSIONS

The method of dusty plasma diagnostics discussed above and based on an analysis of the inhomogeneity of the linear structures of *d* ions seems very attractive. In contrast to the situation in the usual sound method, the *d* ions of a small dust crystal or a linear dust chain have additional degrees of freedom. It gives the possibility to extract additional information from the static response (change of the equilibrium distances between the *d* ions) or the dynamic response (oscillations and waves in inhomogeneous structures). The sounding by small clusters of *d* ions cannot change essentially the plasma parameters (although some distortion of the microfield in the plasma can be stimulated by the traps, which stabilize the *d* clusters). The advantage of static diagnostics is the simplicity of the measurements of the inhomogeneous structure and the simple connection with the parameters of the interaction between *d* ions, their shielding, and the characteristics of rf plasma. The precise theoretical consideration of the dynamical experiments [1,3], which are based on the excitation of the eigenmodes in linear chains and dust crystals, seems a more complicated problem.

We would like to stress that the most general consideration of the equilibrium inhomogeneous configurations of dusty systems can be based on translationally noninvariant solutions of the connected system of kinetic equations for plasmas and Poisson's equation, where the separation between an external field and *d*-*d* interaction is absent. The equilibrium positions for the *d* ions can be found as the points of space where the self-consistent electric field is in balance with gravity. However, this program is too complicated and, as we showed, not necessary for a reasonable theoretical description of the existing experiments.

ACKNOWLEDGMENTS

We are grateful to Dr. A. Melzer for fruitful discussions and private communications about the experimental results. We also would like to thank Dr. H. Thomas and Dr. J. Goree for private communications connected with papers [9–11]. This work has been performed with the support of INTAS Grant No. 96-0617.

[1] S. Peters, A. Homman, A. Melzer, and A. Piel, Phys. Lett. A **223**, 389 (1996).

[2] A. Homman, A. Melzer, S. Peters, and A. Piel, Phys. Rev. E **56**, 7138 (1997).

[3] J.B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).

[4] T.J. Sommerer, W.N.G. Hitchen, R.E.P. Harvey, and J.E. Lawrer, Phys. Rev. A **43**, 4452 (1991).

[5] A. Melzer, V.A. Schweigert, L.V. Schweigert, A. Homman, S. Peters, and A. Piel, Phys. Rev. E **54**, R46 (1996).

[6] V.A. Schweigert, L.V. Schweigert A. Melzer, A. Homman, and A. Piel, Phys. Rev. E **54**, 4155 (1996).

[7] J. Goree (private communication).

[8] A. Melzer (private communication).

[9] G.E. Morfill and H. Thomas, J. Vac. Sci. Technol. A **14**, 490 (1996).

[10] H. Thomas and G.E. Morfill, Nature (London) **379**, 806 (1996).

[11] J.B. Pieper, J. Goree, and R.A. Quinn, Phys. Rev. E **54**, 5636 (1996).